

Design of truss and frame structures using alternative performance indicators

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Abstract: For the design of efficient, sustainable, and safe structures it is often helpful to use alternative performance indicators compared to displacements and stresses for structural evaluation and decision-making. The redundancy matrix provides a detailed and valuable insight into the structural behaviour of statically indeterminate structures. It can be used in the early design phase of truss and frame structures to achieve robustness and to estimate the imperfection sensitivity at an early design stage and independent of the load. The deformation modes and redundancy distribution for trusses and beams will be discussed shortly. Interpretation and application of the redundancy matrix as a basis for alternative structural performance indicators are the main highlights. These can assist in choosing the optimum design in terms of for instance reliability, robustness and sensitivity to imperfections.

Keywords: redundancy matrices, alternative structural performance indicators, numerical design

1 Introduction

One of the overarching goals of the Cluster of Excellence Integrative Computational Design and Construction for Architecture is to improve cooperation between architecture, engineering, and practical manufacturing as so-called “Co-Design” [1]. This should enable a better feedback-loop for the entire design process. To this end, civil engineering is trying to find and apply alternative evaluation methods in order to improve the design process to conserve resources and reduce the material costs. Deformations, strains, and stresses are well known as performance indicators of a structural system, which are load-dependent. For an early assessment of different design variants, alternative indicators can be used, based on the redundancy matrix introduced by Bahndorf [2]. Among others, it describes the distribution of the static indeterminacy within a given structure independent of the applied load, aiding in the identification of potentially critical structural parts.

2 Redundancy in truss and beam structures

In von Scheven et al. [3] a detailed study on the redundancy distribution for trusses and beams is given. The redundancy matrix \mathbf{R} maps the initial elongations $\Delta \mathbf{l}_0$ to the negative elastic elongations $\Delta \mathbf{l}_{el}$.

$$-\Delta \mathbf{l}_{el} = \mathbf{R} \Delta \mathbf{l}_0 \quad \mathbf{R} = \Delta \mathbf{l} - \Delta \mathbf{l}_0 = \mathbf{I} - \mathbf{A} \mathbf{K}^{-1} \mathbf{A}^T \mathbf{C}, \quad (1)$$

where \mathbf{A} is the compatibility matrix, \mathbf{C} is the member stiffness matrix, and \mathbf{I} is the identity matrix. The diagonal entries describe the distribution of static indeterminacy in the structure. For truss elements, the values are between 0 and 1, where $r_{ii} = 0$ describes a statically determinate element with no redundancy. The trace of the redundancy matrix $\text{tr}(\mathbf{R})$ is equal to the degree of static indeterminacy of the system. The extension to beam elements is

straightforward, but instead of only one deformation mode, as in the truss element, elongation, there are three deformation modes due to the additional shear and bending parts. The image of the redundancy matrix $\text{im}(\mathbf{R})$ and the kernel $\text{ker}(\mathbf{R})$ both span a space with special properties, like for instance purely incompatible elongation modes in a structure. For more details see von Scheven et al. [3].

3 Interpretation and Application

The entries of \mathbf{R} can also be interpreted as a measure for the sensitivity to imperfections, since they quantify the constraint that the surrounding structure imposes on an individual element. In order to have a robust and reliable structure with no statically determinate parts, the redundancy of an element should never be zero. On the other hand, small values of redundancy will lead to the small forces in the structure brought on by manufacturing-related imperfections, and might be therefore advantageous during assembly processes in some specific areas. Some examples will be discussed in this domain utilizing the characteristics of the redundancy matrix to obtain an improved design.

4 Conclusion

Alternative performance indicators, based on the redundancy matrix can be used in the design of truss and frame structures to improve overall performance and identify any potentially critical substructures in statically indeterminate structures. It also gives valuable insights into the load-carrying behaviour both on a global structural and on a local element level. There is work in progress with regard to the Co-Design of fiber-reinforced structures, with the aim to develop and optimize large-scale structures in terms of sustainability, assemblability, and increasing efficiency in design and construction.

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