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A bi-fidelity collocation approach for kinetic epidemic models with random inputs

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Abstract: Data uncertainty is certainly one of the main problems in epidemiological modeling. The need for efficient methods capable of quantifying the effects of random inputs on outputs is essential to produce realistic scenarios of the spread of infection and to aim to implement the best control actions. In this work, we consider a bi-fidelity approach to quantify uncertainty in spatially dependent epidemic models. The approach is based on evaluating a high-fidelity model on a small number of samples appropriately selected on the basis of a large number of evaluations of a low-fidelity model, ensuring high computational efficiency and accuracy. In particular, we consider a class of multiscale kinetic transport models for high-fidelity reference and simple discrete-velocity kinetic models for low-fidelity evaluations. Both class of models share the same diffusive behavior and are solved numerically using methods that preserve their asymptotic limits, which permits to obtain stochastic asymptotic-preserving methods. A series of numerical experiments confirms the validity of the approach.

Keywords: Bi-Fidelity methods; Epidemic models; Kinetic transport equations; Stochastic Asymptotic-Preserving schemes; Uncertainty Quantification

1 Introduction

Amongst the various uncertainty quantification techniques, approaches based on non-intrusive stochastic strategies that do not necessarily require a priori knowledge of the probability density function (PDF) of uncertain parameters are particularly interesting in view of comparisons with experimental data, such as in the case of studies on the spread of infectious diseases. In this context, bi-fidelity (or multi-fidelity) methods represent an effective response, thanks to the adoption of control variate techniques based on the appropriate use of low-fidelity (LF) surrogate models, capable of accelerating the convergence of stochastic sampling.

2 Multiscale kinetic transport models for epidemics

We consider the class of multiscale kinetic system presented in [1] to describe an epidemic spread dynamics that takes into account the mobility of individuals and the presence of random inputs. To quantify the effects of these uncertainties following a bi-fidelity approach, the epidemic discrete-velocity kinetic system introduced in [2] is taken as reduced LF model. These models correctly describe the hyperbolic transport dynamics of the movement of individuals over long distances together with the small-scale diffusive nature typical of high-density urban areas, sharing the same diffusive limit. This allows for sufficient similarity in the model's random parameters space and enables the effectiveness of the following approach.

3 Asymptotic-preserving bi-fidelity collocation method

Considering a random input $z \in \Omega \subset \mathbb{R}$, the bi-fidelity collocation approach makes use of the solution of a computationally cheap LF model $u^{LF}(z)$ to effectively inform the selection of a small number of representative collocation points in the random parameter space. Thus, to construct accurate approximations of high-fidelity (HF) solutions $u^{HF}(z)$ evaluating the computationally expensive HF model only in a small number of selected samples, we build an inexpensive surrogate $u^{BF}(z)$ of the HF model in the following non-intrusive manner [3, 4]: $u^{HF}(z) \approx u^{BF}(z) = \sum_{k=1}^{N} c_k(z) u^{HF}(z_k)$, $z_k \in \gamma_N$, where N is the number of the selected collocation points in the parameter space $\gamma_N = \{z_1, \ldots, z_N\}$. Here, these coefficients are acquired from the LF model, considering $c_k(z) \approx c_k^{LF}(z)$. Finally, to solve the systems at each collocation point, an asymptotic-preserving IMEX Finite Volume scheme that works uniformly in all regimes is adopted to obtain an efficient stochastic asymptotic-preserving method.

4 Conclusions

To examine the performance of the proposed methodology, several benchmark tests for different regimes are considered, which fully confirm the effectiveness of the approach.

References

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