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Uncertainty quantification of differential equations with random parameters: methods and applications

Scientific area: Computational Applied Mathematics

As numerical simulations become more and more spread in science and engineering, we are often confronted with the fact that we might have an imperfect knowledge of the components/parameters of the mathematical model that we wish to simulate.

Often these models consist of partial or ordinary differential equations (PDE and ODE, respectively) where coefficients, forcing terms, boundary and initial conditions or even the shape of the computational domain are possibly not known precisely. This uncertainty might stem from measurement errors, lack of knowledge, or intrinsic random variability of the phenomenon under consideration.

It then becomes crucial to endow the predictions obtained by numerical simulations with information about the level of confidence on such predictions, taking into account the uncertainty in the model components. This can be done by assuming that the parameters of the ODE/PDE can be described as random variables or random fields, and then “propagating their variability through the equation”, i.e., estimating statistical quantities such as expected value, variance, quantiles, or probability density function of the solution of the PDE/ODE at hand, or of functionals thereof. This kind of analysis is called uncertainty quantification (UQ), and more specifically, forward UQ. A closely related analysis is the so-called inverse UQ, whose goal is to narrow the uncertainty on the parameters of the ODE/PDE as soon as (noisy) measurements of the solution become available. More advanced applications include the design of experiments, i.e., devising optimized measurement campaigns to reduce the uncertainties in the PDE/ODE, and optimization under uncertainty, whose goal is to compute the optimal design of a system with uncertain parameters such that its response is optimized in a suitable statistical sense, e.g., minimizing the variance of the solution or maximizing its expected value.

UQ is thus a cross-disciplinary field, that builds on elements of numerical analysis, statistics, and computational sciences, that has gained considerable traction in the past decade, due to the increased computational power of computers and to the availability of more sophisticated algorithms. Indeed, while the most basics tool to perform UQ are simple (yet robust) methods such as Monte Carlo, more advanced methodologies such as multi-level/multi-fidelity sampling method, sparse grids collocation, stochastic Galerkin, reduced basis, proper orthogonal/generalized decomposition, low-rank tensor representation, and, recently, machine-learning techniques such as neural networks have proven to be the key to enable large-scale applications of UQ analysis, and to make UQ tools available to a broader audience. In this session, we aim at bringing together young investigators with different backgrounds (applied math, computational and data sciences, engineering, numerical analysis, statistics) working in the area of UQ. Both methodological and application-oriented presentations are welcome.

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